

# Naked Singularities as Particle Accelerators

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We investigate here the particle acceleration by naked singularities to arbitrarily high center of mass energies. Recently it has been suggested that black holes could be used as particle accelerators to probe the Planck scale physics. We show that the naked singularities serve the same purpose and probably would do better than their black hole counterparts. We focus on the scenario of a self-similar gravitational collapse starting from a regular initial data, leading to formation of a globally naked singularity. It is seen that when particles moving along timelike geodesics interact and collide near the Cauchy horizon, the energy of collision in the center of mass frame will be arbitrarily high, thus offering a window to Planck scale physics.

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Recently, an interesting observation was made by Bannados, Silk and West [1], that black holes can accelerate infalling colliding particles to arbitrarily high energy in the center of mass frame around the horizon of an extremal Kerr black hole, provided certain restrictive conditions were imposed on the angular momenta of the particles. While this mechanism was criticized for being a very fine tuned one [2], [3], a number of further works have recently investigated the process for different black holes, e.g. in the context of an extremal charged spinning black hole [4], non-extremal rotating black holes [5, 6], non-rotating charged black-holes [7], and so on. It is also claimed that this may be a generic property of rotating black holes in a model independent way [8].

We show here that the divergence of center of mass energy of colliding particles is a phenomenon not only associated with black holes, but also with naked singularities which are outcome of a continued gravitational collapse of a massive star. We consider here a self-similar spherically symmetric spacetime. There have been several numerical and analytical investigations of self-similar gravitational collapse for different matter fields satisfying reasonable energy conditions, such as dust [9], ideal fluids with non-vanishing pressures [10], massless scalar fields [11, 12], and such others, leading to the formation of naked singularities in gravitational collapse from a regular initial data. Naked singularity formation in collapse models has been investigated in detail in recent years (see e.g. [13] and references therein), and a wide variety of physically relevant situations, self-similar or otherwise, are included.

Whether or not naked singularities would occur in the real world we live in, is an unanswered question till this date. However, considering the very many results of gravitational collapse scenarios that lead to a naked singular-

ity final fate for collapse, we may assume that naked singularities could occur in various physical circumstances such as the final fate of a massive star, when it collapses at the end of its life cycle on exhausting its internal nuclear fuel. It would be then of much interest to investigate the consequences of their formation, taking into account the possible quantum gravity effects these may cause [14], and possible connection with the very highly energetic astrophysical phenomena such as the gamma ray bursts and those related to the active galactic nuclei.

We show here that the naked singularities forming in gravitational collapse could provide us a window into the new Planck scale physics, even far away from the actual singularity. This is because, unlike the case of a black hole, the particles could go very close to a naked singularity and then emerge with very high velocities near the Cauchy horizon that the naked singularity created. For definiteness, we focus here on self-similar models but the results may hold good in more general class of collapses.

A self-similar spacetime is characterized by the presence of a vector field  $\xi$ , known as the homothetic killing vector, which satisfies

$$L_{\xi}g_{\mu\nu} = 2g_{\mu\nu} \quad (1)$$

where  $L_{\xi}$  stands for a Lie derivative along the vector field  $\xi$ , and  $g_{\mu\nu}$  is the spacetime metric. A spherically symmetric self-similar spacetime geometry in the  $(t, r, \theta, \phi)$  coordinates can be written as,

$$ds^2 = -e^{2\nu(X)}dt^2 + e^{2\psi(X)}dr^2 + r^2S(X)^2d\Omega^2 \quad (2)$$

where  $\xi = t\partial_t + r\partial_r$  is the homothetic vector field and  $X = t/r$  is the self-similarity variable,  $d\Omega^2$  being the metric on a two-sphere. It is easily verified that the metric (2) satisfies (1). For a given matter field, one can write down and solve the Einstein field equations, which reduce to ordinary differential equations in this case, to obtain the metric functions  $\nu(X), \psi(X)$ , and  $S(X)$ . As mentioned, the exact solution to the Einstein field equations for dust as a matter field [9], and numerical solution with

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an ideal fluid with linear equation of state [10] have been obtained and are shown to admit the naked singularity as collapse final state. We keep our analysis here general and model independent, without explicitly referring to any of the particular solutions mentioned above, though clearly our conclusions apply to each of these cases.

The naked singularity as final outcome of collapse is characterized by existence of families of outgoing null and timelike geodesics, which terminate in the past at the singularity and in future they reach a faraway observer in the spacetime. The first null geodesic that comes out from the singularity is the Cauchy horizon in the spacetime. Before considering the particle collisions near the Cauchy horizon, we first note certain general features of a self-similar spacetime admitting naked singularity in collapse, developing from a regular initial data.

The Ricci scalar for (2) is of the form,

$$\mathfrak{R} = \frac{f(X)}{r^2} \quad (3)$$

where  $f(X)$  is a function of self-similarity variable alone, which can be written explicitly in terms of the metric functions  $\nu(X), \psi(X)$ , and  $S(X)$ . If  $f(X_0)$  is a finite, non-zero number, approaching  $(t=0, r=0)$  along the curve  $t = X_0 r$  makes the Ricci scalar diverge as  $r \rightarrow 0$ . Hence  $t=0, r=0$  is the spacetime singularity. The full singularity curve is given by  $\frac{t}{r} = X = X_s$ , where  $X_s$  is a smallest positive number for which  $f(X_s) \rightarrow \infty$  and the Ricci scalar (3) diverges. It is known that various other curvature scalars also diverge at the singularity  $t=0, r=0$ , which is a strong curvature singularity in the spacetime.

We are interested in the situation where gravitational collapse starts from a regular data specified on an initial spacelike hypersurface. Therefore  $f(X)$  is finite for  $X < 0$ , i.e.  $t < 0$ . The regular initial data implies the regularity of center  $r=0$  for  $t < 0$ , i.e. as  $X \rightarrow -\infty$ . Hence  $f(X) \sim \frac{1}{X^2}$  as  $X \rightarrow -\infty$ , so that  $\mathfrak{R} \sim \frac{1}{t^2}$ , and the center  $r=0$  is regular for  $t < 0$ . The structure of such a collapse spacetime is depicted in Fig 1.

The apparent horizon in the spacetime is given by  $g^{\mu\nu} \partial_\mu R \partial_\nu R = 0$ , where  $R = rS(X)$ , and has the equation,

$$e^{2\psi} S_X^2 = e^{2\nu} (S - X S_X)^2 \quad (4)$$

The solution  $X = X_{ah}$  to (4) gives the apparent horizon curve  $t/r = X_{ah}$ . If  $X_s < X_{ah}$  then it would be possible to have outgoing null and timelike curves from the singularity curve to infinity, which would be globally visible. When the central singularity  $(t=0, r=0)$  is visible, the equation of outgoing radial null geodesics  $ds^2 = 0$  from the same is given by  $(\frac{dt}{dr} = e^{\psi-\nu})$ . Since close to the  $(t=0, r=0)$ ,  $\frac{dt}{dr} \approx \frac{t}{r} = X$ , we get,

$$X^2 e^{2\nu} = e^{2\psi} \quad (5)$$

close to the origin. However, it is easily verified in this case that this also represents a null geodesic away from

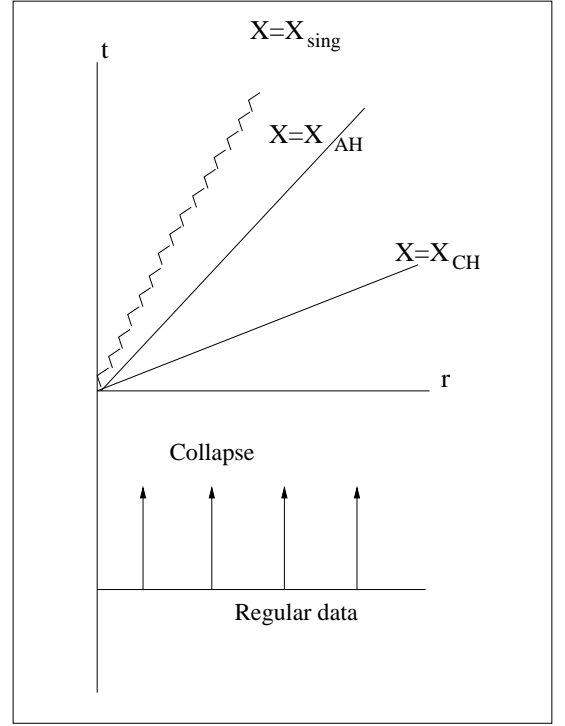


FIG. 1: The general structure of a self-similar spherically symmetric spacetime admitting naked singularity. The singularity curve, apparent horizon, and the Cauchy horizon are given by  $X = X_{sing}, X_{ah}, X_{ch}$  respectively.

the origin. There would be outgoing radial null geodesics to infinity from  $(t=0, r=0)$  if there exists a positive solution  $X = X_c$  to (5), such that  $X_c < X_{ah}$ , and the singularity would then be naked. We then have a situation where a self-similar spherically symmetric gravitational collapse from a regular initial data on an initial hypersurface gives rise to a naked singularity, and there are outgoing null geodesics to infinity from  $(t=0, r=0)$ . The equation (5) also describes the Cauchy horizon, which is the first outgoing null geodesic from the naked singularity.

Consider now a test particle of mass  $m$ , following a timelike radial geodesic on this spacetime. Let  $\lambda$  be an affine parameter along the geodesic and  $U^\mu$  be the velocity vector. Then using the geodesic equation  $U^\mu{}_{;\nu} U^\nu = 0$  and normalization condition  $U^\mu U_\mu = -1$ , we get,

$$\frac{d}{d\lambda} (U^\mu \xi_\mu) = -1 \quad (6)$$

Using (2) we obtain

$$(U^\mu \xi_\mu) = -e^{2\nu} t U^t + e^{2\psi} r U^r = C - \lambda \quad (7)$$

where  $C$  is an integration constant. The normalization condition for velocity is

$$-e^{2\nu} (U^t)^2 + e^{2\psi} (U^r)^2 = -1 \quad (8)$$

The nonvanishing components of velocity are,

$$U^t = \frac{(X \pm e^{2\psi} Q)(C - \lambda)}{r(e^{2\psi} - e^{2\nu} X^2)}; U^r = \frac{(1 \pm X e^{2\nu} Q)(C - \lambda)}{r(e^{2\psi} - e^{2\nu} X^2)} \quad (9)$$

where

$$Q = \sqrt{e^{-2\psi-2\nu} + \frac{r^2 e^{-2\psi-2\nu} (e^{2\psi} - e^{2\nu} X^2)}{(C - \lambda)^2}}$$

Here the positive and negative signs correspond to outgoing and ingoing geodesics respectively [13][15]. It can be verified easily that the above satisfy (7),(8).

Consider now two particles of identical mass  $m$  (for simplicity), which collide very close to the Cauchy horizon, the first being an outgoing one, coming from a close vicinity of the naked singularity along a radial timelike geodesic from the singularity, and the second being an infalling one that crosses the Cauchy horizon to fall inwards. From (5), since  $e^{2\psi} - X^2 e^{2\nu} \approx 0$ ,  $Q \approx e^{-\psi-\nu}$ , so the components of velocities (outgoing  $U_+^\mu$  and ingoing  $U_-^\mu$ ) of particles can be written as,

$$U_+^t = \frac{B_1 e^{-\nu}}{r(e^\psi - X e^\nu)}; U_+^r = \frac{B_1 e^{-\psi}}{r(e^\psi - X e^\nu)} \quad (10)$$

$$U_-^t = \frac{-B_2 e^{-\nu}}{r(e^\psi + X e^\nu)}; U_-^r = \frac{B_2 e^{-\psi}}{r(e^\psi + X e^\nu)} \quad (11)$$

where  $B_i = C_i - \lambda_i$ . We focus here only on the outgoing radial timelike geodesics in the region of spacetime beyond the Cauchy horizon where  $e^{2\psi} - X^2 e^{2\nu} < 0$ . Since  $U^t > 0$ , we must have  $B_1 < 0, B_2 < 0$ .

The energy of collision in the center of mass frame is given by [1],

$$E_{cm} = \sqrt{2}m\sqrt{1 - 2g_{\mu\nu}U_1^\mu U_2^\nu} \quad (12)$$

The center of mass energy computed from ingoing and outgoing particles of mass  $m$ , colliding close to the Cauchy horizon, from (11),(10),(12) is then given by,

$$E_{cm} \approx \frac{2\sqrt{2}m}{r} \sqrt{\frac{B_1 B_2}{e^{2\nu} X^2 - e^{2\psi}}} \quad (13)$$

This would be arbitrarily large, depending on how close is the point of collision to the Cauchy horizon. A point to be noted here is that in general the collision event could be far away from the singularity.

The advantage of the scenario considered here is, we already have the outgoing timelike geodesics existing from an arbitrary vicinity of the singularity, by the very structure of the spacetime geometry. These then meet the infalling particles arbitrarily close to the Cauchy horizon. The key difference from its black hole counter-part is, we consider here the collision between an outgoing and an

ingoing particle, rather than two ingoing ones as in the black hole case.

Such outgoing particles from a close neighborhood of singularity, which travel close to the Cauchy horizon, could arise in various ways. One can consider the region of spacetime before the formation of singularity, and the ingoing geodesics starting from a faraway region, after passing through the regular center would emerge as outgoing geodesics. The outgoing particles close to the Cauchy horizon would be the ingoing geodesics, which just missed the singularity ( $t = 0, r = 0$ ), and emerge as outgoing particles. Such ingoing particles in collapse may miss the singularity if they had a small angular momentum or due to the small perturbations in geometry. The region around singularity would be fuzzy, and dominated by quantum gravity effects. When non-perturbative semi-classical modifications to the classical evolution dynamics are taken into account from quantum gravity, the fuzzy region around what would have been classically a naked singularity, is shown to be "super-repulsive" for arbitrary matter field configurations in the late regime [16],[14]. Hence the particles emerging out of this region may get boosted up in energy significantly and would travel close to the Cauchy horizon for a long time. These particles, accelerated by the singularity, would then meet the ingoing particles close to the Cauchy horizon, and the collision would occur at arbitrarily large center of mass energies.

It would be interesting to examine whether this result would generalize to many of the collapse scenarios where a naked singularity develops even when self-similarity is relaxed. The divergence of colliding particles near Cauchy horizon could then be a generic phenomenon associated with naked singularities. In that case one could probe the Planck scale physics in a region away from the actual singularity, but near the Cauchy horizon. For instance, one may construct a model of an inhomogeneous dust collapse of a finite matter cloud (Fig.2), the exterior being a Schwarzschild metric. For a wide class of initial conditions of the initial density and velocity profiles, the collapse would give rise to a naked singularity, and eventually a black hole. Before the apparent horizon engulfs the surface of the star, turning it into a black hole, the Planck scale physics would be visible at the surface, as the Cauchy horizon hits it. This might trigger unknown channels of reactions between elementary particles and could have impact on our understanding of phenomena such as the active galactic nuclei, provided they can be modeled this way. Since dark matter could be modeled by dust, the situation described above may be applicable to the gravitational collapse of halos of dark matter, and might have astrophysical implications.

It is interesting to note here that the Cauchy horizon for black holes, which also coincides with their inner horizon, exhibits an instability termed as mass inflation [17]. The particles colliding near the Cauchy horizon of

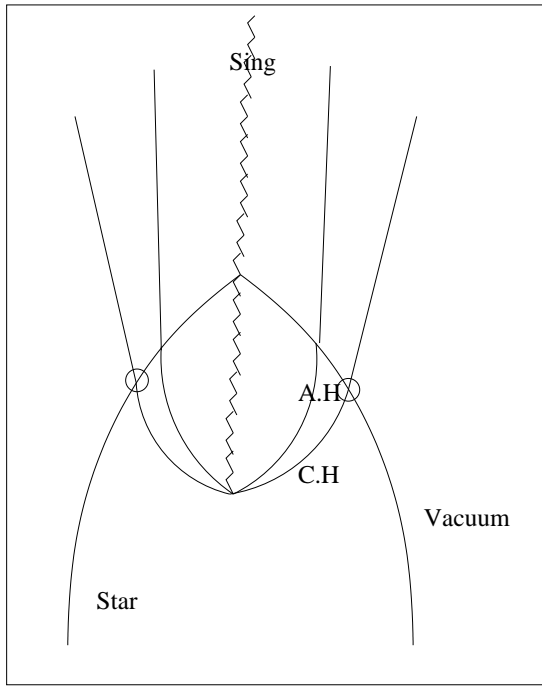


FIG. 2: The matter cloud undergoes a collapse to form a naked singularity. A.H. is apparent horizon, C.H. is Cauchy horizon, Sing is singularity. The circle depicts region on the surface of the star where Planck physics would be visible.

naked singularity and exhibiting divergence in the center of mass energy frame may indicate another form of instability for the Cauchy horizon. We have treated here the colliding particles as test particles, ignoring gravitational radiation and the backreaction on the spacetime.

There are key differences between the particle acceleration by black holes and naked singularities. The divergence of center of mass energy in case of a black hole is near its event horizon. The new particles created in such a collision would either enter the horizon and would never be detected, or if they escape the black hole they

could be infinitely redshifted for an asymptotic observer to see them, and for whom the process requires an infinite amount of time [3]. Hence it is very unlikely that black holes as super-colliders would be useful to probe Planck scale physics. However, since the collisions with divergent center of mass energies occur near the Cauchy horizon in the case of naked singularities, it would be possible to detect the particles, as the redshift and time required for this process would be finite. Hence, as a probe of Planck scale physics naked singularities could do a better job compared to their black hole counterparts.

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